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ScienceDirect
Magnetic Resonance Imaging 24 (2006) 1341–1347
MAGNETIC RESONANCE IMAGING

Structure-specific magnetic field inhomogeneities and its effect on the correlation time

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Received 24 January 2006; accepted 25 August 2006

Abstract

We describe the relationship between the correlation time and microscopic spatial inhomogeneities in the static magnetic field. The theory takes into account diffusion of nuclear spins in the inhomogeneous field created by magnetized objects. A simple general expression for the correlation time is obtained. It is shown that the correlation time is dependent on a characteristic length, the diffusion coefficient of surrounding medium, the permeability of the surface and the volume fraction of the magnetized objects. For specific geometries (spheres and cylinders), exact analytical expressions for the correlation time are given. The theory can be applied to contrast agents (magnetically labeled cells), capillary network, BOLD effect and so forth.

Keywords: Correlation time; Susceptibility effects; Relaxation effects

1. Introduction

Magnetic field inhomogeneities affect the motion of spins from which they are surrounded. Especially in the case of biological tissue, diffusion has an important influence on clinically relevant parameters. The inhomogeneous field may be produced by magnetically labeled cells, deoxygenated red blood cells, capillary network in the heart or brain and iron-rich cells in the brain, or it is purposefully produced by contrast agents. Usually, the transverse relaxation time T_2* is used to obtain information about tissue parameters like the density of labeled cells, oxygenation states of blood or vascular network.

The parameter T_2* is based on an exponential magnetization decay of the FID of the form ~exp($-t/T_2$*). Taking the non-Gaussian character of spin dephasing into account [1], a more accurate consideration of the relaxation process leads to the result in which the exact time course of the magnetization decay of the FID signal is dependent on the relaxation time in the motional narrowing limit T_2,0 and the correlation time τ describing the correlation function of the local frequencies

$$M(t) = \exp\left(-\frac{t}{T_{2,0}}\right)\exp\left[\frac{\tau}{T_{2,0}}\left(1-e^{-t/\tau}\right)\right], \tag{1}$$

where the relaxation rate in the motional narrowing limit is given by $R_{2,0} = T_{2,0}^{-1} = \tau\langle\omega^2(\vec{r})\rangle$. For example, in the case of human blood, the magnetization decay caused by the inhomogeneous field around the erythrocytes can be described by a non-Gaussian one [2]. Now, the question of how the correlation time is dependent on the local frequency distribution and the volume fraction of the magnetized objects arises. Spherical objects were first studied in Ref. [3], where the correlation function has been found. The exact form of the correlation function was extensively studied by Sukstanskii and Yablonskiy [4] and Jensen and Chandra [5]. They demonstrated that the time dependence of the correlation function is not exponential but algebraic. Sukstanskii and Yablonskiy [4] gave a simple expression for the correlation function, which takes into account the real distribution of the inhomogeneous magnetic field created by the objects of arbitrary geometry. In contradiction to them, to arrive at simple expressions, we assume the correlation function to

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doi:10.1016/j.mri.2006.08.005

be a simple exponential decay of the form ~exp($-t/\tau$). In this article, we present a rigorous derivation of the correlation time τ only in dependence of the local frequency distribution $\omega(\vec{r})$ and the assumed boundary conditions. A simple expression for the correlation time is given, which is easy to implement numerically, and for the special case of spheres and cylinders, we give an analytical expression.

2. General approach

We consider a compact, homogeneous magnetized body in an external magnetic field. We observe the trajectory of diffusing spins. Homogeneous diffusion properties outside the magnetic body are assumed; that is, we neglect the potential diffusion restrictions by membranes or other structures. Instead of considering diffusion in the whole tissue, we only focus on the mean relaxation volume per magnetized object (Fig. 1). The form of this relaxation volume is dependent on the form of the magnetized object; that is, in the case of a magnetized sphere with radius R_C, the relaxation volume is the space between two concentric spheres with the radii R_C and R and the volume fraction of material is $\eta=R_C^3/R^3$. In analogy, in the case of cylinders, the relaxation volume is the space between two concentric cylinders with radii R_C and R and volume fraction $\eta=R_C^2/R^2$. In this case, Fig. 1 shows a cross-sectional view of the cylinder. Diffusion is then restricted to the space between two concentric objects with radii R_C and R; that is, periodic boundary conditions are assumed at the outer surface of the relaxation volume. It is important to emphasize that the sphere with radius R is only a mathematical boundary and has nothing in common with the cell nucleus or cell membrane. The rationale for this restriction and its mathematical implications for evaluation of diffusion have already been discussed in detail [6].

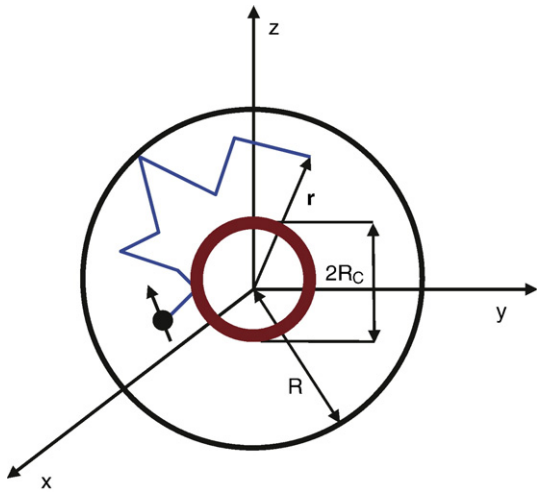

Fig. 1. Geometry of the magnetized object and the coordinate system. One implication of this is that reflectory boundary conditions have to be assumed for the trajectories of a nuclear spin at R; that is, trajectories of spins that pass through the surface of a relaxation sphere into the next one must be replaced by symmetric trajectories that are located within the original relaxation sphere.

The two-point correlation function of the stochastic field fluctuations to which a spin is subjected is defined as

$$K(t)=\int_V d^3\vec{r}\int_V d^3\vec{r}_0\,\omega(\vec{r})p(\vec{r},\vec{r}_0,t)p(\vec{r}_0)\omega(\vec{r}_0), \tag{2}$$

where $p(\vec{r},\vec{r}_0,t)$ is the probability density of finding a spin at point $\vec{r}$ after time t with the initial ($t=0$) position $\vec{r}_0$ and $p(\vec{r}_0)$ specifies the probability density function of the equilibrium distribution. In our case, the latter is identical with the spin density, which we assume to be homogeneous; that is,

$$p(\vec{r}_0)=\frac{1}{V}, \tag{3}$$

where V is the dephasing volume (in the case of spheres, the volume between concentric spheres of radius R_C and R). The probability $p(\vec{r},\vec{r}_0,t)$ obeys the Smoluchowski equation $\partial_t p(\vec{r},\vec{r}_0,t)=\nabla\vec{j}(\vec{r},\vec{r}_0,t)$, where the corresponding flux is given by $\vec{j}(\vec{r},\vec{r}_0,t)=D\nabla p(\vec{r},\vec{r}_0,t)$. We assume free diffusion of spins within the boundaries $R_C\le r\le R$; then, the probability $p(\vec{r},\vec{r}_0,t)$ is simply the Green's function of the diffusion equation where D is the diffusion coefficient

$$\frac{\partial}{\partial t}p(\vec{r},\vec{r}_0,t)=D\nabla^2 p(\vec{r},\vec{r}_0,t) \tag{4}$$

or

$$p(\vec{r},\vec{r}_0,t)=e^{tD\nabla^2}\delta(\vec{r}-\vec{r}_0). \tag{5}$$

While reflectory boundary conditions are assumed on the outer surface of the relaxation volume (in the three-dimensional case, the sphere with radius R, or in the two-dimensional case, the cylinder with radius R), for the inner surface, we distinguish between reflectory and radiation boundary conditions. In the case of an impermeable inner surface, the trajectories of diffusing spins are continued by a symmetric one if the spin reaches the inner surface; that is, we impose reflectory boundary conditions

$$\frac{\partial p(\vec{r},\vec{r}_0,t)}{\partial r}\Big|_{r=R_C}=0=\frac{\partial p(\vec{r},\vec{r}_0,t)}{\partial r}\Big|_{r=R}. \tag{6}$$

The reflectory boundary conditions at the outer surface R enclose our system and prevent any nuclear spin from escaping to infinity. Therefore, the corresponding flux at this surface vanishes: $\vec{j}(R,t)=0$. If the exchange of water molecules between the relaxation volume and the magnetized object is taken into account, one imposes the radiation boundary conditions at the inner surface

$$D\frac{\partial p(\vec{r},\vec{r}_0,t)}{\partial r}\Big|_{r=R_C}=kp(\vec{r},\vec{r}_0,t)\Big|_{r=R_C}, \tag{7}$$

where k is a microscopic rate constant describing the permeability of the inner surface [7].

Insertion of the probability density given in Eqs. (5) and (3) into the definition of the correlation function Eq. (2) results in

$$K(t) = \frac{1}{V} \int_V d^3\vec{r}\, \omega(\vec{r}) e^{tD\nabla^2} \omega(\vec{r}). \tag{8}$$

In general, the correlation function $K(t)$ does not exhibit a single exponential decay as it is often assumed [8], which hampers the simple determination of the correlation time. However, a proper definition of the correlation time is to define it as the mean relaxation time of the correlation function; that is, according to [9],

$$\tau = \int_0^\infty dt \frac{K(t)}{K(0)}. \tag{9}$$

A commonly used approximation for the correlation function is $K(t) \approx K(0) e^{-t/\tau}$. As shown by Jensen and Chandra [5], when the diffusion is unrestricted, $K(t)$ is better approximated by an algebraic function. Insertion of Eq. (8) into Eq. (9) yields

$$\tau = \frac{1}{\langle \omega^2(\vec{r}) \rangle DV} \int_V d^3\vec{r}\, \omega(\vec{r}) \left[-\frac{1}{\Delta} \right] \omega(\vec{r}), \tag{10}$$

where the variance of the local magnetic field is given by

$$\langle \omega^2(\vec{r}) \rangle = K(0) = \frac{1}{V} \int_V d^3\vec{r}\, \omega^2(\vec{r}) \propto \eta \delta\omega^2, \tag{11}$$

with η as the fraction of the total volume of magnetic material inside the tissue. The relation $\langle \omega^2(\vec{r}) \rangle \propto \eta\delta\omega^2$ is in complete agreement with the more general Eq. (1) given in Ref. [4]. Using the general expression in Eq. (10), we are able to find a relation between the correlation time τ, the volume fraction η, the radius of the perturber R and the diffusion coefficient D. This will lead to a relation also mentioned in Eq. (15) in the publication of Stables et al. [10].

3. Specific geometrical objects

The general expression (Eq. (10)) obtained above is applied to two shapes of the magnetized object: spheres and cylinders. To solve the integral (Eq. (10)), we first have to look for a function $f(\vec{r})$, which satisfies the equation

$$\nabla^2 f(\vec{r}) = -\omega(\vec{r}). \tag{12}$$

In the case of spheres and cylinders, we can find this function analytically. For more sophisticated objects, the Laplace equation has to be solved numerically. Furthermore, the function $f(\vec{r})$ has to fulfill the reflectory boundary condition (Eq. (6))

$$\frac{\partial f(\vec{r})}{\partial r}\Big|_{r=R} = 0 \tag{13}$$

at the outer surface of the relaxation volume. In the case of radiation boundary conditions at the inner surface, the function $f(\vec{r}) = f_{\mathrm{rad}}(\vec{r})$ obeys

$$D \frac{\partial f_{\mathrm{rad}}(\vec{r})}{\partial r}\Big|_{r=R_C} = k f_{\mathrm{rad}}(r = R_C), \tag{14}$$

and in the case of reflectory boundary conditions at the inner surface, $f(\vec{r}) = f_{\mathrm{ref}}(\vec{r})$ obeys

$$D \frac{\partial f_{\mathrm{ref}}(\vec{r})}{\partial r}\Big|_{r=R_C} = 0. \tag{15}$$

3.1. Spheres

The inhomogeneous field around a magnetized sphere is that of a magnetic dipole; that is, in spherical coordinates (r,θ,φ) [11],

$$B(\vec{r}) = \mu_0 \frac{\Delta M}{3} R_C^3 \frac{3\cos^2\theta - 1}{r^3}, \tag{16}$$

where $\Delta M = M_C - M_T$ is the difference between the magnetization of the magnetized sphere M_C and the surrounding tissue M_T. Introducing the characteristic equatorial frequency shift $\delta\omega = \gamma |B(r=R_C, \theta=\pi/2)| = \gamma\mu_0\Delta M/3$, where γ is the gyromagnetic ratio, we have

$$\omega(\vec{r}) = \delta\omega R_C^3 \frac{3\cos^2\theta - 1}{r^3}. \tag{17}$$

Following the argumentation of Stables et al. [10], we assume that the effects of the neighboring capillaries are negligible. Introducing this local frequency in Eq. (11), we find

$$\langle \omega^2(\vec{r}) \rangle = \frac{4}{5} \eta\, \delta\omega^2. \tag{18}$$

The result, $K(0) = \langle \omega^2(\vec{r}) \rangle = \frac{4}{5}\eta\,\delta\omega^2$, is the same as Eq. (18) from Jensen and Chandra [5] or Eq. (8) from Stables et al. [10]. To solve the Laplace equation (Eq. (12)), for the field of a dipole, we use the expression

$$f(\vec{r}) = g(r) \cdot \left(3\cos^2\theta - 1\right). \tag{19}$$

If we apply the Laplace operator ∇^2 in spherical coordinates to the function, we arrive at a differential equation for the radial function $g(r)$: $r^2 \partial_r^2 g(r) + 2r\partial_r g(r) - 6g(r) = -\delta\omega R_C^3/r$. This Euler's type of differential equation can be transformed into an inhomogeneous differential equation with constant coefficients by using the substitution $r = e^t$. Finally, we find for the radial function:

$$g(r) = \frac{A}{r^3} + Br^2 + \frac{\delta\omega R_C^3}{6r}, \tag{20}$$

where the integration constants A and B can be determined by taking the boundary conditions at the inner and outer surfaces into account. Using Eq. (19) for solving the Laplace equation

(Eq. (12)), we can reduce the expression for the correlation time (Eq. (10)) to the simple form

$$\tau = \frac{3}{D\delta\omega(1-\eta)} \int_{R_C}^{R} dr \frac{g(r)}{r}. \tag{21}$$

In the case of radiation boundary conditions at the inner surface, using Eq. (14), we find the integration constants:

$$A_{\text{rad}} = \frac{\delta\omega}{6} R_C^5 b \quad \text{and} \quad B_{\text{rad}} = \frac{\delta\omega}{4}\left(b\eta^{5/3} + \frac{\eta}{3}\right) \tag{22}$$

with the abbreviation

$$b = \frac{2\frac{D}{kR_C}(\eta-1)+\eta+2}{6\frac{D}{kR_C}(\eta^{5/3}-1)-3\eta^{5/3}-2}. \tag{23}$$

In the case of reflectory boundary conditions at the inner surface, using Eq. (15), we find the integration constants:

$$A_{\text{ref}} = \frac{\delta\omega R_C^5}{18}\frac{\eta-1}{1-\eta^{5/3}} \quad \text{and} \tag{24}$$

$$B_{\text{ref}} = \frac{\delta\omega}{12}\left(1+\frac{\eta-1}{1-\eta^{5/3}}\right).$$

Using the abbreviation

$$d_i = \sum_{j=0}^{i} \eta^{\frac{j}{3}}, \tag{25}$$

we arrive at an expression for the correlation time considering radiation boundary conditions

$$\tau_{\text{rad}} = \frac{R_C^2}{2D}\left[\frac{1}{4} + \frac{3}{4d_2} + b\left(\frac{1}{3} + \frac{3}{4}\eta^{5/3}\right)\right], \tag{26}$$

and in the case of an impermeable core, the correlation time is only dependent on the volume fraction, the sphere radius and the diffusion coefficient

$$\tau_{\text{ref}} = \frac{R_C^2}{8D}\left[\frac{3}{d_2} + \frac{5d_2}{9d_4}\right]. \tag{27}$$

A Taylor expansion in the term η leads to a simple relationship between correlation time and the volume fraction η:

$$\tau_{\text{rad}} = \frac{R_C^2}{D}\left(1 - \frac{1}{3}\frac{1+\frac{D}{kR_C}}{1+3\frac{D}{kR_C}} - \frac{3}{4}\sqrt[3]{\eta}\right), \tag{28}$$

and

$$\tau_{\text{ref}} = \frac{R_C^2}{D}\left(\frac{4}{9} - \frac{3}{8}\sqrt[3]{\eta}\right) \tag{29}$$

for the correlation time in the case of an impermeable core.

3.2. Cylinders

In the case of cylinders, we have to take into account their orientation to the external magnetic field. The frequency shift is dependent on the angle θ between the axis of the cylinder and the direction of the external magnetic field B_0. In analogy to the frequency around a sphere (Eq. (17)), we find the local frequency to be

$$\omega(\vec{r}) = \delta\omega_\theta R_C^2 \frac{\cos 2\phi}{r^2}, \tag{30}$$

where the frequency shift at the surface of the cylinder is given by $\delta\omega_\theta = \gamma\Delta\chi/2B_0\sin^2\theta$, with the gyromagnetic ratio γ and the susceptibility difference $\Delta\chi$ between the cylinder and the surrounding tissue [12]. In Fig. 1, we see in this case a cross-sectional view of the cylinder (suppression of one dimension by omitting, e.g., the x-axis) and two-dimensional polar coordinates (r,ϕ) are used. The variance of this field directly follows from Eq. (11) as

$$\langle\omega^2(\vec{r})\rangle = \frac{1}{2}\eta\,\delta\omega_\theta^2, \tag{31}$$

which is in agreement with Stables at al. [10]. Considering the form of the angle-dependent part of the frequency (Eq. (30)) in analogy to Eq. (19), we use in the case of cylinders the expression $f(\vec{r}) = g(r)\cdot\cos 2\phi$, which leads to the differential equation $r^2\partial_r^2 g(r) + r\partial_r g(r) - 4g(r) = -\delta\omega_\theta R_C^2$ and its resolution $g(r) = Ar^2 + B/r^2 - \delta\omega_\theta R_C^2/4$. If we arrange the same steps as described in the example above, we arrive at an analogous expression for the correlation time:

$$\tau_{\text{rad}} = \frac{R_C^2}{4D}\left(\frac{1+\eta}{\frac{2D}{kR_C}(\eta^2-1)-\eta^2-1} - \frac{\ln\eta}{1-\eta}\right) \tag{32}$$

for radiation boundary conditions and

$$\tau_{\text{ref}} = -\frac{R_C^2}{4D}\frac{\ln\eta}{1-\eta} \tag{33}$$

for reflectory boundary conditions, which is agreement with the result of Bauer et al. [13].

4. Applications

4.1. Diffusion regimes

Based on the picture of spins diffusing around a field inhomogeneity, two frequency scales characterizing the underlying relaxation mechanism are present. The dynamic frequency scale $1/\tau$ characterizes the stochastic process of diffusion using the correlation of moving spins. The magnetic frequency scale characterized by the equatorial frequency $\delta\omega$ specifies the relaxation caused by the local field inhomogeneity. Based upon the relative magnitude of these two characteristic frequencies, five diffusion regimes can be introduced to describe the relaxation process:

1. $1/\tau \gg \delta\omega$, motional narrowing regime
2. $1/\tau > \delta\omega$, fast diffusion regime
3. $1/\tau = \delta\omega$, intermediate regime
4. $1/\tau < \delta\omega$, slow diffusion regime
5. $1/\tau \gg \delta\omega$, static dephasing regime.

Analytical expressions for the relaxation time T_2* exist for each diffusion regime. The application of such an analytical expression assumes knowledge of the underlying diffusion regime. Knowing the characteristics of tissue, that is, the characteristic magnetic field shift $\delta\omega$ and the dynamic frequency $1/\tau$, leads to a decision about which diffusion regime should be applied. Based on qualitative arguments, Yablonskiy and Haacke [14] gave the following criterion for the static dephasing regime:

$$\frac{(\bar{r}/2)^2}{D}\delta\omega\frac{\eta}{2d}\gg 1, \tag{34}$$

where $\bar{r}/2$ is the characteristic distance of magnetic field change and $d=1$, 2 and 3 for one-, two- and three-dimensional diffusion. In the case of spherical particles, the static dephasing will hold if

$$\frac{R_C^2}{D}\delta\omega\frac{\sqrt[3]{\eta}}{6}\gg 1. \tag{35}$$

Using our results for the volume-fraction-dependent correlation time, we are able to deviate similar expressions based on a direct mathematical analysis. In the case of spheres, we obtain from Eq. (29) for the static dephasing regime ($\tau\delta\omega\gg 1$) the criterion

$$\frac{R_C^2}{D}\delta\omega\left(\frac{4}{9}-\frac{3}{8}\sqrt[3]{\eta}\right)\gg 1. \tag{36}$$

In analogy, for cylinders, we can obtain from the correlation time (Eq. (33)) a similar criterion for the validity of the static dephasing regime.

To illustrate the application of the correlation time, we consider cells that are labeled with magnetized spheres (SPIO). For a typical volume fraction of magnetized spheres in a voxel, we consider the value $\eta\approx 10^{-7}$ [15]. As a typical value for the radius of a sphere, we consider a commercial SPIO agent with a diameter of $2R_C=60$ nm [16]. For the diffusion coefficient in biological tissue, we assume a value of $D=1$ μm^2/ms. Using Eq. (29), we obtain for the correlation time the value $\tau=0.4$ μs, corresponding to a characteristic frequency of $1/\tau=2.5$ MHz.

The correlation time around cylinders can be used to quantify the diffusion process in the myocardium [13]. In this case, the volume fraction is equivalent to the regional blood volume $\eta\approx 0.05$. The typical diameter of a capillary in the myocardium is $2R_C=5.5$ μm [17]. Using the same value for the diffusion coefficient as above, we obtain from Eq. (33) the correlation time $\tau=6$ ms, corresponding to a characteristic frequency of $1/\tau=168$ Hz.

4.2. Relaxation rates

Spin dephasing caused by magnetic field inhomogeneities influences the magnetization decay. This magnetization decay can be described by the Bloch–Torrey Equation [18]. Signal formation in the static dephasing regime [14] was extensively studied in the case for spheres [19] and cylinders [20]. To apply the methods demonstrated above, we derive the relaxation rate in the motional narrowing limit. In this diffusion regime, signal loss is predominantly caused by incoherent dephasing due to spin diffusion in an inhomogeneous magnetic field. As opposed to signal that is lost by static dephasing, this signal loss cannot be recovered using spin-echo sequences.

The frequency fluctuation caused by the diffusive process, which is characterized by the inverse of the correlation time, is much greater than the frequency fluctuation caused by the magnetic sphere, $1/\tau\gg\delta\omega$. For this case, the relaxation rate can be obtained from the simple relation

$$R_2^*=\tau\langle\omega^2(\vec{r})\rangle, \tag{37}$$

where $\langle\omega^2(\vec{r})\rangle$ for spheres and cylinders is given in Eqs. (18) and (31), respectively. In Fig. 2, we compare the relaxation rates R_2* dependent on the volume fraction η for spheres and cylinders.

In the case of small volume fraction, we obtain from Eqs. (37) and (29) a simple expression for the relaxation rate of a suspension of spheres

$$R_2^*=\frac{16}{45}\eta\,\delta\omega^2\frac{R_C^2}{D}, \tag{38}$$

which is in agreement with the results of Moiny et al. [21]. Jensen and Chandra [5] determined the relaxation rate in the case of a permeable core:

$$R_2^*=\frac{8}{25}\eta\,\delta\omega^2\frac{R_C^2}{D}. \tag{39}$$

Combining this result with the general expression (Eq. (37)), we obtain the correlation time for a permeable core

$$\tau=\frac{2}{5}\frac{R_C^2}{D}, \tag{40}$$

which is in agreement with previously obtained results [22].

To quantify the range of validity of this low-density approximation, we first consider the case of capillaries. We assume a regular arrangement; that is, in the cross-sectional

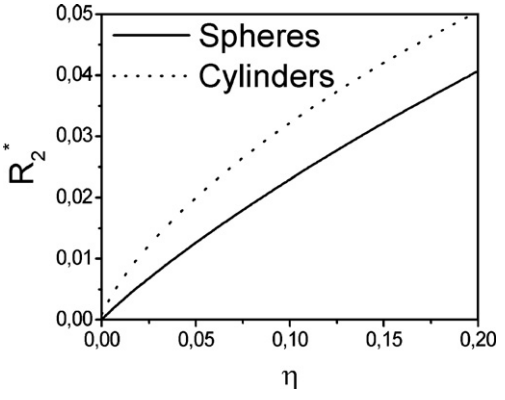

Fig. 2. Relaxation rate in the case of spheres and cylinders, obtained from Eq. (37). In both cases, we assumed $R_C^2/D=1$ for the prefactor.

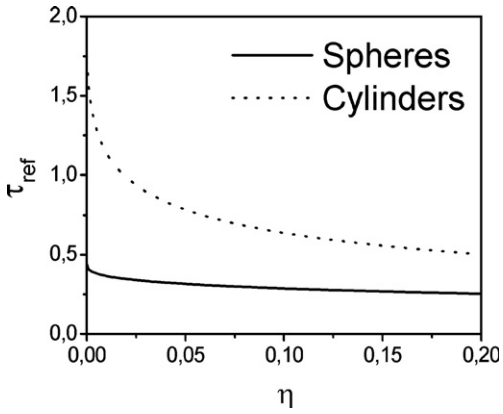

Fig. 3. Correlation time in the case of spheres and cylinders. The continuous line is obtained from Eq. (27), and the dotted line is obtained from Eq. (33). In both cases, we assumed $R_C^2/D=1$ for the prefactor.

view, the capillary is in the center of a hexagon and has six nearest neighbors. Each hexagon with side length a and area $A=\frac{3}{2}\sqrt{3}\,a^2$ is in our model (see Fig. 1), replaced by a circle with radius R and the same area $A=\pi R^2$. Therefore, the distance between two capillaries is $2a=2\sqrt{2\pi/(3\sqrt{3})}R$. The effects of neighboring capillaries can be neglected if the characteristic frequency $\delta\omega_\theta$ at the surface of a capillary is much higher than the frequencies caused by the six surrounding capillaries with distance $2a$. Using Eq. (30), we obtain the inequality $\delta\omega_\theta \gg 6\delta\omega_\theta R_C^2/(2a)^2$, which leads to an estimation for the volume fraction: $\eta \ll 4\pi/\left(9\sqrt{3}\right)=0.81$. In the three-dimensional case, each sphere is in the center of a rhombic dodecahedron (the Wigner–Seitz cell of the fcc Bravais lattice) and is surrounded by 12 nearest neighbors. Each rhombic dodecahedron with side length s and volume $V=16s^3/3\sqrt{3}$ is in our model (see Fig. 1), replaced by a sphere with radius R and the same volume $V=4\pi R^3/3$. Therefore, the distance between two spheres is $2\sqrt{2/3}s$. The effects of neighboring spheres can be neglected if the characteristic frequency $\delta\omega$ at the surface of a sphere is much higher than the frequencies caused by the 12 surrounding spheres with distance $2\sqrt{2/3}s$. Using Eq. (17), we obtain the inequality $\delta\omega \gg 12\delta\omega R_S^3/\left(2\sqrt{2/3}s\right)^3$, which leads to an estimation for the volume fraction: $\eta \ll \sqrt{2}\pi/9=0.49$.

5. Summary and conclusions

Based on a simple geometry of magnetized objects in an external magnetic field and using the theory of the mean relaxation time, we gave an expression (Eq. (10)) for the correlation time τ describing the diffusion process around these objects. This correlation time is a function of a typical diameter of the object, the diffusion coefficient of the surrounding medium, the volume fraction of magnetized material and, in the case of a permeable object, the permeability of the surface. It is expected that this concept, because of its mathematical simplicity, will prove fruitful for many further applications.

Within the diffusion space, we only considered the interaction of a nuclear spin with the magnetic field of one sphere or cylinder; that is, the field contribution of the neighboring spheres or cylinders was neglected. In a tissue where the field inhomogeneities have a small volume fraction and a weak surrounded field, this assumption may be justified and the low-density approximations given in Eqs. (28) and (29) are valid.

For the special case of spheres and cylinders, analytical expressions for the correlation time are given, which can be used for characterizing diffusion regimes or relaxation rates. To demonstrate the influence of the shape of the magnetized object onto the correlation time, in Fig. 3, we compare the volume fraction dependence of the correlation time for spheres and cylinders.

Acknowledgments

This work was supported by the Schering Stiftung and the Deutsche Forschungsgemeinschaft - Sonderforschungsbereich 688 “Mechanismen und Bildgebung von Zell-Zell-Wechselwirkungen im kardiovaskulären System.”

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